Seventeenth International Conference on Application
of Mathematics in Technical and Natural Sciences
24–29 June 2025, Albena

BOOK OF ABSTRACTS

Euro-American Consortium for Promoting the Application
of Mathematics in Technical and Natural Sciences

Edited by Michail Todorov

Sofia • 2025

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A Mathematical Model of CDK1 and APC Interactions in Cancer Cells: Exploring the Impact of Zingerone on Cell Proliferation

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The cell cycle is a fundamental process through which cells grow and divide. Disruptions in cell cycle regulation can lead to uncontrolled cell growth, a hallmark of cancer. Among the key regulators of the cell cycle are Cyclin-Dependent Kinase 1 (CDK1) and the Anaphase-Promoting Complex (APC), which drive cells through the cell cycle.

Cancer, a leading cause of death globally, has attracted substantial research attention, including mathematical approaches that offer quantitative analyses across various cancer types and treatment strategies. In particular, a foundational model described in describes how CDK1 and APC are expected to interact with each other, resulting in oscillatory dynamics. From this model, one may devise strategies to slow cell duplication, a critical factor in cancer progression. The primary focus of our research is to investigate how reducing the cyclin synthesis rate using the natural compound Zingerone can inhibit melanoma cell viability. The existence of a limit cycle is demonstrated by analysing the foundational model, which describes the activation and inactivation dynamics of CDK1 and APC. A detailed qualitative analysis is performed to validate the system's domain, and the approximate period of the limit cycle is calculated. Thereafter, simplifying the model allows for explicit mathematical solutions to easily be found and parameter constraints applied to the model. From this simplified model, the period of the limit cycle is again determined, shedding light on cell cycle oscillations under various conditions.

Experimental cell viability data was collected at multiple time points and under different Zingerone concentrations. Although some measurement error is inherent, fitting theoretical functions to experimental data helps manage variability, confirming that Zingerone effectively slows cancer cell progression. This underscores the value of mathematical modeling and natural treatment methods in cancer research.

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Modeling of the Goodwin Business Cycle Using Gamma Distributed Induced Investment

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The dynamics of the temporal behavior of income $y(t)$ in the Goodwin business cycle model can be analyzed using both neutral delay differential equation [1,2] and integro-differential equation

$$\varepsilon \frac{dy(t)}{dt} + sy(t) = \int_{-\infty}^{t} y(t-\zeta)\varphi\left(\frac{dy(\zeta)}{d\zeta}\right) + A(t) \quad (1)$$

with a continuous distributed delay kernel of induced investments $\varphi = \varphi\left(\frac{dy(t)}{dt}\right)$ in the form of gamma distribution [3]

$$f(t) = g_a^n(t) = \frac{a^n t^{n-1} e^{-at}}{(n-1)!}, \; n = 1, 2, \ldots .$$

In this work, we study the influence of the gamma distribution shape parameter n on the dependence $y(t)$. If for small n the solutions have the form of long periodic limit cycles, then for large values of n there are smooth limit cycles with a small period close to the characteristic delay time in induced investments. The threshold values of m for various parameters included in the Goodwin equation (1) will be found numerically.

References

[1] R. M. Goodwin (1951) *Econometrica* **19**, 1–17.
[2] R. H. Strotz, J.C. McAnulty, and J.B. Naines (1953) *Econometrica* **21**, 390–411.
[3] A. Matsumoto, F. Szidarovszky, "Delay differential nonlinear economic models," in G. Bischi, C. Chiarella, L. Gardini (eds) Nonlinear Dynamics in Economics, Finance and Social Sciences (Springer, Berlin, Heidelberg 2010).

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Linear Systems and Integral Equations: A Stochastic Approach

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We develop stochastic algorithms for solving linear systems of high dimensions and Fredholm integral equations. The algorithms are based on Monte Carlo simulations of suitably defined Markov Chains. We aim to examine an algorithm which works for both for integral equations and linear systems.

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Transport Safety – Methodology, Quantitative and Qualitative Assessment of the Link Between Meteorological Conditions and Safety

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Security by its very nature is the protection of interests related to the life of the individual, the organization and society as a whole from existing threats - real and potential. Any system is complex and conditions, as well as external and internal factors, lead to negative processes with respect to the safety management system. The sense of security is an integral part of human life and a strong factor that determines concepts and projects in various fields, including transport. The understanding of the nature of safety, the sustainability of the same is universal but should be defined for a specific area in the transport service. In order to improve road safety, it is necessary to systematize the optimization methodology into certain sequential steps and actions that lead to the reduction of wrong decisions. The authors propose a methodology for road safety system management. The safety performance functions can provide an estimate of the total accident frequency for the baseline conditions, for any section, for any road. Critical point analysis techniques are mainly classified by identifying potential safety issues. A significant part of the work of identifying accident hotspots is the use of various identification and selection methodologies. The safety management methodology

aims to minimize crashes within minimum resources, reduce crashes resulting in fatalities, and finally, high-traffic roads are prioritized to ensure traffic safety. It should be noted that weather conditions influence the occurrence of accidents on roadways to some extent.

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Impact of Inequality in Income Distribution on Population Happiness

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This study examines and tests the hypothesis that higher inequality has a negative effect on the happiness of the population. Different conditions are analyzed, including countries with equal or similar volumes of GDP per capita. Data is collected for 42 European countries, including those similar to Bulgaria. The information is subjected to logical experiments and empirical tests through two regression analyses and calculation of correlations, with a detailed interpretation of the results.

Key words: inequality, economics, income, society, happiness

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Comparison Principle for Reaction-Diffusion Systems with Delays and Application in SEIR Spatial models

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Comparison principle, existence and uniqueness of the weak solutions for cooperative reaction-diffusion systems with delays are considered. Spatial SEIR models in Epidemiology are described by such systems. Introducing delays in the system incorporates the incubation period and time for recovery in the system itself. Use of reaction-diffusion model allows more precise account of the inhomogeneity of the density of the population -in the cities population is dense, while in the countryside it is not.

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An Analytic Investigation of the Emergence of a Secondary Diffusion in a Thin Film Model

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The Bevilacqua-Galeão Model of Anomalous Diffusion introduces two fluxes: a primary flux that follows Fick's law of diffusion, representing the fraction of particles undergoing classical diffusion, and a secondary flux modeled by a fourth-order differential term, which accounts for retention phenomena. We investigate the "analytic emergence" of this secondary flux by treating the model as a perturbation of the heat equation, which describes classical diffusion. Rather than applying traditional perturbation methods, we employ the powerful framework of modern group analysis to study this problem. Specifically, by utilizing approximate symmetries - first introduced by Baikov, Gazizov, and Ibragimov - we are able to derive an analytic expression for the emergent secondary diffusion as a perturbation of the classical diffusion process.

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Vibration Control and Parameter Optimization of a Negative Stiffness Dynamic Vibration Absorber

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With the growing complexity of modern engineering systems, interdisciplinary integration has become essential for addressing key technological challenges, particularly in the field of vibration control, which has broad application prospects. Integrating applied mathematics, engineering mechanics, and materials science enables the

effective application of vibration theory to real-world problems, which is crucial for ensuring the safety and precision of aerospace, transportation, and high-end instrumentation systems. In this study, the vibration control and optimization design of a negative stiffness dynamic vibration absorber (DVA) with a parallel structure are investigated. A systematic methodology combining classical optimization theory and intelligent algorithms is adopted. Based on the fixed-point theory, analytical expressions for the optimal tuning frequency ratio, stiffness ratio, and approximate damping ratio are derived, providing theoretical support for subsequent multi-parameter optimization via intelligent algorithms. To refine the key parameters, a particle swarm optimization algorithm is employed, resulting in approximately equal peak amplitudes at the two resonant frequencies in the amplitude-frequency response curve. Comparative simulations with several typical DVAs demonstrate that, under harmonic excitation of the primary system, the proposed model leads to a significant reduction in resonance peaks, enhances system stability, and improves vibration attenuation performance. The presented work offers both theoretical and algorithmic support for parameter optimization and the practical engineering application of DVAs incorporating negative stiffness elements.

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Application of Hilbert Transform to the Analysis of Electromagnetic Oscillations Beats and Alpha Rhythm Spindles

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The concept of an analytical signal and the Hilbert transform, applied to a narrowband signal are considered. The application of the Hilbert transform to a signal used in the beating method is considered. A numerical calculation of the instantaneous frequency of a signal, which is the sum of two harmonic oscillations of close frequencies, is performed. The development of methods for analyzing electroencephalograms is an important and relevant task. The importance of finding new methods for studying alpha'rhythm spindles is shown. The alpha'rhythm of a healthy awake person is considered. Hilbert transform is applied to each spindle and the instantaneous frequency dependencies are determined. It is shown that the spindles have the beating form and the calculated instantaneous frequency dependencies of spindles confirm this.

Keywords: Narrowband signal, Hilbert transform, beating method, alpha rhythm spindles

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Exact Solutions to the 1D φ^4 Ginsburg-Landau Model Subject to Neumann-infinity Boundary Conditions

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In this work, we consider a thermodynamic system with the shape of a thin film. It is assumed that at a given temperature and external ordering field, the behavior of the regarded system is described by a scalar order parameter that minimizes the one-dimensional counterpart of the standard φ^4 Ginzburg-Landau Hamiltonian and meets the so-called Neumann-infinity boundary conditions. An analytic representation of the extremals of this variational problem is given by means of Weierstrass elliptic functions. Then, depending on the temperature and ordering field, we find the respective minimizers, obtain the phase diagram and Casimir force in the temperature-field plane.

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Approximate Solution of Non-Homogeneous Biharmonic Problems in Non-Smooth Domains

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Let $\Omega \subset \mathbb{R}^2$ be a simply connected domain with piecewise smooth boundary Γ, $x = (x_1, x_2) \in \mathbb{R}^2$ and ∂_j, $j = 1, 2$ refers to the partial derivative in the x_j direction. In the talk, we are going to discuss the approximate solution of the following problem:

$$\begin{aligned} &\Delta^2 U(x) = f(x), && x \in \Omega, \\ &U^+(t) = g(t), && t \in \Gamma, \\ &(\partial_{\mathbf{n}} U)^+(t) = h(t), && t \in \Gamma, \end{aligned} \tag{1}$$

where $\Delta = \partial_1^2 + \partial_2^2$ is the Laplace operator, $\mathbf{n}(t) = (n_1(t), n_2(t))^\intercal$ the unit normal vector field on the boundary Γ, $\partial_{\mathbf{n}} = \sum_{j=1}^2 n_j \partial_j$ the normal derivative, and $\varphi^+(t)$ denotes the trace of function $\varphi\colon \Omega \to \mathbb{R}$ on the boundary Γ. Under natural conditions on the functions f, g, h, the problem (1) is translated into two boundary value problems (BVPs) for the homogeneous biharmonic equation with non-homogeneous boundary conditions. Each of this BVPs is reduced to the Sherman-Lauricella equation

$$\omega(t) + \frac{1}{2\pi i}\int_\Gamma \omega(\tau)\, d\ln\left(\frac{\tau - t}{\overline{\tau} - \overline{t}}\right) - \frac{1}{2\pi i}\int_\Gamma \overline{\omega(\tau)}\, d\left(\frac{\tau - t}{\overline{\tau} - \overline{t}}\right) = F(t),\ t \in \Gamma, \qquad (2)$$

with specific right-hand sides F. The Eqs. (2) are solved by spline Galerkin methods. It is shown that the methods are stable if and only if certain operators from an algebra of Toeplitz operators are invertible. These operators do not depend on the shape of the boundary Γ but only on the magnitude of its angles. They have a complicated structure and, at present, efficiently verifiable conditions for their invertibility are not known. Nevertheless, numerical studies suggest that for angles $\alpha \in (0.1\pi, 1.9\pi)$, all such operators are invertible.

The solutions of the Sherman-Lauricella equations are then used o determine numerical solutions of the biharmonic problem (1), and we also study the errors of this method. It is worth noting that for piecewise smooth domains, the approximate solutions of (1) do not keep the convergence order of the approximate solutions of (2), but halve it. This is in sharp contrast to the case of smooth boundaries.

The approach used has a number of advantages, one of which is its applicability to biharmonic problems in any domain with a simple piecewise smooth boundary, whereas popular finite difference and finite element methods are mainly restrained to problems in polygonal domains. Moreover, the method can be extended on biharmonic problems in multiple connected domains with piecewise smooth boundary.

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New Analytical Formulaes for the Weierstrass Invariants of Elliptic Integrals in the Weierstrass Form Depending on the Modulus Parameter *q* of the Integral in the Legendre Form

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An elliptic integral in the Legendre form will be transformed into an elliptic integral in the Weierstrass form. Two different representations of the Weierstrass form of the elliptic integral have been found, one of which is the s.c. method of four-dimensional uniformization. A new formulae for the Weierstrass invariants will be found, depending in a complicated manner on the modulus parameter q of the integral in the Legendre form.

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First-Principles Study of Radiation-Induced Defects in Silicon Solar Cells and Annealing Using Density-Functional Theory Simulation

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Radiation-induced point defects significantly degrade the performance of silicon solar cells, particularly in space and high-energy environments. These defects disrupt charge transport, introduce recombination centers, and reduce carrier lifetimes. Thermal annealing is a widely recognized approach to mitigate radiation damage; however, a detailed atomistic understanding of defect behavior under annealing conditions remains incomplete. In this study, we employ density-functional theory (DFT) simulations to investigate the thermally driven evolution of representative point defect complexes in both p-type and n-type silicon, with particular focus on boron-related (BiOi, BiBiOi, BiOiOi), carbon-related (CiOi, CiOiOi), and vacancy-type (VO, VB, VV) defects. We performed dsimulation across a temperature range of 50°C to 250°C, with analysis focusing on structural distortions, bond length variation, and Fermi energy shifts.

Our results demonstrate defect-specific thermal responses. Boron-related complexes exhibit significant structural reconfiguration during annealing. Notably, the Bi-Oi bond in the BiOi complex shortens by approximately 17% (from 1.5073 Å at room temperature to 1.2450 Å at 250°C), accompanied by elongation of the Bi-Si bond and a substantial upward Fermi level shift of +0.16eV. These trends suggest potential electronic destabilization at elevated temperatures, indicating that boron-related defects are thermally tunable and may partially anneal at moderate temperatures (100°C). Higher temperatures ($>$ 150°C) may be necessary for full passivation or transformation. These findings are consistent with the experimental studies of radiation-induced defects in Si.

Conversely, carbon-based defects exhibit thermal resilience, with minimal structural changes (Ci-Oi bond variation $<$ 0.4 Å) and nearly constant Fermi levels across the annealing window. Similarly, vacancy-related defects display structural and electronic stability, reinforcing their role as persistent damage centers unless targeted by more aggressive thermal or chemical interventions. Vacancy-type defects remain structurally and electronically robust throughout the annealing process. These findings are consistent with experimental studies of radiation-induced defects and their recovery upon annealing in Si.

The results highlight the potential of tailored annealing strategies for mitigating radiation damage in silicon solar cells, providing practical guidance for optimizing thermal processing conditions. These findings offer mechanistic insights into the annealing dynamics of intrinsic and extrinsic point defects in crystalline silicon. They highlight the critical role of impurity type and bonding environment in governing defect stability under thermal treatment. Furthermore, this work establishes a foundation for future studies investigating extended temperature ranges, longer annealing durations, and experimental validation to bridge computational predictions with real-world device performance. From a device engineering perspective, this study supports the use of annealing protocols tailored to specific defect types, optimizing the recovery of radiation-degraded solar cells while informing doping and material design strategies for next-generation, radiation-hardened photovoltaic technologies.

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Improved Analytical Solution for Turbulent Flow in Channel

A. Fedoseyev

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The approximate analytical solution for turbulent flow in channel was proposed in Fedoseyev [3], that described the mean turbulent flow velocity as a superposition of the parabolic and superexponential solutions. The Alexeev Hydrodynamic Equations (AHE) were used as the governing equations to describe the turbulent flow, [1]. The AHE have new terms, temporal and spatial fluctuations, compared to the Navier-Stokes equations (NSE). These new terms have a timescale multiplier τ, and the AHE reduce to the Navier-Stokes equations when $\tau \to 0$.

In this study, the improved analytical solution formula is proposed, that provides a better agreement with experimental data. The maximum discrepancy of analytical solution and experimental data has reduced from 5% to 2% for Reynolds number below 100,000 and from 10% to 4% for Reynolds number up to 35,000,000, comparing with experimental data from publication starting of legacy paper by [4] (Prandtl group, 1932), to [6], [7], [2] and recent research of [5].

References

[1] B.V. Alexeev (1994) The generalized Boltzmann equation, generalized hydrodynamic equations and their applications, *Phil. Trans. Roy. Soc. London, A* **349** 417–443.

[2] C.W.H. Van Doorne, J. Westerweel (2007) Measurement of laminar, transitional and turbulent pipe flow using Stereoscopic-PIV, *Experiments in Fluids* **42**(2).

[3] A. Fedoseyev (2023) Approximate Analytical Solution for Turbulent Flow in Channel, *J. Phys.: Conf. Ser.* **2675**, 012011.
[4] J. Nikuradse (1932), Laws of turbulent flow in smooth pipes, (English translation) NASA TT F-10: p. 359 (1966).
[5] S. Pasch, R. Leister *et al* (2023) Measurements in a Turbulent Channel Flow by Means of an LDV Profile Sensor. Flow, Turbulence and Combustion, *Springer Nature 2021.*
[6] T. Wei, W.W. Willmarth (1989) Reynolds-number effects on the structure of a turbulent channel flow, *J. Fluid Mech.* **204**, 57–95.
[7] M.V. Zagarola, A.J. Smits, S.A. Orszag, V. Yakhot (1996) Experiments in high Reynolds number turbulent pipe flow, AIAA Paper 96-0654, AIAA Meeting Papers, January 1996.

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Turbulent Flow in Channel: Experiments versus Numerical Simulation and Analytical Solution

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The experiments of turbulent flow in channels provide data that are used to validate numerical simulations, improve theoretical models, and bring new insights into the fundamental mechanisms of turbulence that may not be captured by existing theoretical models.

In this study, we use Alexeev Hydrodynamic Equations (AHE) to describe the turbulent flow, which takes into account the finite particle size, [1]. The AHE have new terms, temporal and spatial fluctuations, with a timescale multiplier τ. The AHE reduce to the Navier-Stokes equations when $\tau \rightarrow 0$.

The numerical solution of AHE, using the finite element method, has been obtained for a turbulent flow In channel. The experimental data by [4], [3] are compared to the obtained numerical solution and to analytical solution [2] for Reynolds numbers Re = 5940, 14000, 29828, 45552, 79164. The results of comparisons are discussed.

References

[1] B.V. Alexeev (1994) The generalized Boltzmann equation, generalized hydrodynamic equations and their applications. Phil. Trans. Roy. Soc. London, A. 349, 417-443.
[2] A. Fedoseyev (2023) Approximate Analytical Solution for Turbulent Flow in Channel, *J. Phys.: Conf. Ser.* **2675**, 012011.

[3] S. Pasch, R. Leister *et al* (2023), Measurements in a Turbulent Channel Flow by Means of an LDV Profile Sensor. Flow, Turbulence and Combustion, *Springer Nature 2021.*
[4] T. Wei, W.W. Willmarth (1989), Reynolds-number effects on the structure of a turbulent channel flow *J. Fluid Mech.* **204**, 57–95.

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Critical Reynolds number for Transition to Turbulence of Viscous Flow in Channel

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The onset of turbulent flow in the channel is analyzed by numerical continuation methods, using the transverse velocity equation, Fedoseyev (2024) in formulation of Alexeev Hydrodynamic Equations (AHE) as governing equation, Alexeev (1994). The laminar flow in the channel is characterized by zero transverse velocity and may exist up to Reynolds number of $\mathrm{Re}_1 = 100,000$, as shown by experiments. However, the onset of the turbulent flow occurs at much smaller Reynolds number Re_c . The turbulent flow can evolve to an additional secondary turbulent flow at Reynolds number of Re_2 about $10,000$, and starting at this point, three potential solutions can exist, *etc.*

The graph of turbulent flow evolution and the transverse velocity magnitude versus Reynolds number are provided.

References

[1] B.V. Alexeev (1994) The generalized Boltzmann equation, generalized hydrodynamic equations and their applications, *Phil. Trans. Roy. Soc. London A* **349**, 417–443.
[2] A. Fedoseyev (2024) Analytical solution for transverse velocity of turbulent flow in channel, *J. Phys.: Conf. Ser.* **2910**, 012013

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Modeling a Valid Work Schedule using Integer Optimization over a Non-Convex Set

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This study proposes a mixed-integer linear programming (MILP) model for automated driver shift scheduling, addressing the complex challenge of balancing legal regulations (minimum rest intervals, consecutive rest periods), technical constraints (driver-vehicle compatibility), and worker preferences. Using the Big M method, nonlinear dependencies are transformed into linear constraints while preserving solution accuracy. Implemented in MATLAB, the model is validated with real-world data, demonstrating its effectiveness in balancing operational demands, legal requirements, and employee preferences, as well as adaptability to dynamic changes. Applicable in transport, logistics, healthcare, and manufacturing, the approach offers practical tools to reduce costs, enhance service reliability, and improve working conditions. The scientific contribution includes a novel MILP framework for multicriteria scheduling and resource management strategies that integrate regulatory compliance with social considerations. The results highlight the role of mathematical modeling in sustainable human resource management.

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A Multicriteria Optimization Approach for Investment Project Funding

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Investment projects frequently undergo evaluation based on diverse criteria, including economic, social, environmental, and additional relevant factors. Each project is assigned points reflecting its performance in these criteria. Due to limited budgets, not all qualified projects can be fully funded. This creates a challenging decision-making situation, where funding organizations need to select the best projects while maximizing both the number of ap-proved projects and their overall quality. Additionally, practical funding de-cisions often result in partial rather than

full financial support, requiring frac-tional resource allocation. This paper proposes a multicriteria mixed-integer optimization model ex-plicitly designed to allocate limited financial resources among competing in-vestment projects by maximizing both the number of approved projects and their cumulative quality scores. The proposed approach allows partial fund-ing, thereby enhancing flexibility and ensuring efficient utilization of availa-ble resources. The applicability and benefits of the proposed model are demonstrated through illustrative examples.

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Thermodynamic Properties of Quantum Spin Systems: Scaling and Disorder

G. G. Grahovski

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In this talk, we will discuss thermodynamic properties of spin-1/2 XY antiferromagnetic Heisenberg ladders by means of the stochastic series expansion quantum Monte Carlo technique. This includes the thermal properties of the specific heat, uniform and staggered susceptibilities, spin gap, and structure factor.

We will present numerical simulations, probed over a large ensemble of random realizations in a wide range of disorder strengths r: from the clean ($r = 0$) case, up to the diluted ($r \to 1$) limit, and for selected choices of number of legs L_y per site.

Our results show some interesting phenomena, like the presence of crossing points in the temperature plane for both the specific heat and uniform susceptibility curves which appear to be universal in r.

Acknowledgement. Based on a joint work with Erol Vatansever and Nikolaos Fytas [1].

References

[1] E. Vatansever, G. G. Grahovski and N. G. Fytas (2024) Thermodynamic properties of disordered quantum spin ladders, *Eur. Phys. J. B* **97**, `https://doi.org/10.1140/epjb/s10051-024-00669-5`, `arXiv.2312.12834`.

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Reparametrization Invariant Scaling Symmetry as Resolution of the Li-7 Problem within the BBNS

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A possible resolution of the ^{7}Li problem within the Standard Model Big-Bang Nucleosynthesis (SBBN) is presented. The key idea originates from the application of the Scale-Invariant Vacuum (SIV) paradigm to the BBN. The outcome is $\chi^2 < 0.04$ fit to the observed primordial abundances of ^{4}He, D/H, ^{3}He/D, and fit of $\chi^2 \approx 1$ when including ^{7}Li/H observations. The results are obtained and compared to the known SBBN values by utilizing the publicly available PRIMAT code. The resolution of the ^{7}Li problem requires SIV-guided deviation from the local thermal equilibrium during BBN, such that the thermal energy of matter and radiation scale differently with respect to the SIV-conformal factor λ during the BBN epoch. This may be viewed as conformal symmetry breaking due to cooling of plasma and the properties of matter. As such, the framework may be of relevance to the problem of the nuclear fusion as well. The deduced baryon matter content is $\Omega_b \approx 12\%$ for unbroken SIV and $\Omega_b \approx 38\%$ for partially broken SIV, but with $\lambda < 1$ in both cases, which signals preference for Reparametrization Invariant Symmetry Scaling (RISS) over the conventional SIV viewpoint. Applying the RISS paradigm results in $\lambda > 1$ and $\Omega_b \approx 10\%$ with clear departure of n_T away from the naive SIV suggested value. In all the cases where the ^{7}Li problem is resolved, the baryon content is significantly higher than the usually accepted value of $\Omega_b \approx 4.9\%$ within the ΛCDM.

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Bifurcation of Periodic Solutions for a Multi-degree-of-freedom model of Resonator Metamaterials

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Metamaterials, as artificially designed periodic structures, exhibit significant application potential in fields such as smart sensing, vibration control, and stealth technology, owing to their capabilities in manipulating wave propagation characteristics. Among them, resonator metamaterials achieve tunable broadband bandgap

regulation through localized resonance mechanisms, while bifurcation theory provides a pathway for dynamically manipulating wave propagation. Therefore, the bifurcation mechanisms of resonator metamaterial models and the intrinsic correlation mechanisms between bifurcations and band gaps remain to be further investigated.

In this paper, we investigate the bifurcation of periodic solutions for a class of multi-degree-of-freedom resonator metamaterials. By leveraging the Melnikov vector function and the new notations for block matrices, the existence conditions and number of periodic solutions of the system are obtained. Numerical simulations are conducted to reveal the geometric configurations of its periodic orbits. The results demonstrate that optimizing geometric parameters can effectively regulate dynamical behaviors. This study provides a theoretical framework for analyzing the nonlinear dynamics of resonator metamaterials, offering theoretical foundations for their engineering implementations in bandgap control of resonator metamaterials.

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Investigation of the Vortices Couple Motion in a Stratified Fluid

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A mathematical study of the problem of the vortex couple motion in stratified fluid has been conducted. Salinity is chosen as a stratifying component. The vortex velocity in a wide range of Reynolds and Froude numbers was investigated. The task is described by the Navier-Stokes equations in the Boussinesq approximation. To solve the problem, the SMIF method is used, the finite different scheme of which has such properties as the second order of approximation on spatial variables, minimal scheme dissipation and dispersion, solving wave processes are a property of monotony.

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Probability Metrics for Kolmogorov-Arnold Net (KAN)

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Probability metrics are used to evaluate the approximation accuracy of Kolmogorov-Arnold Networks (KAN) for complex (random) functions encountered in biomedicine and healthcare. The KAN is based on the Kolmogorov-Arnold representation theorem which states that any multivariate continuous function can be represented as a sum of continuous univariate functions composed with a set of linear functions. Unlike deep neural networks (DNNs), which utilize multiple layers for approximation, the KAN framework achieves universal approximation with a shallow network architecture. That structure leads sometimes to interpretable models, making it easier to understand the contribution of different inputs. KAN models hold great potential in approximating continuous functions (including random functions), but a robust framework for their evaluation in biomedical contexts is lacking. On the other hand, probability metrics are measures used to quantify the similarity, divergence, or distance between two probability distributions and could serve as such evaluation framework. These metrics are essential when comparison of distributions is critical for analysis, inference, or model evaluation.

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Synergizing XGBoost and Propensity Score Matching (PSM) for Early Detection and Causal Insight in Cardiac Rejection

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This study will develop a methodological framework integrating predictive and causal inference techniques to analyze a high-dimensional dataset with one record per unit, drawn from the THORACIC subset of the OPTN STAR File (through September 30, 2024). We will employ XGBoost, optimizing a regularized log-loss function, via second-order gradient boosting to model a binary outcome across heart transplant patient characteristic variables, assessing performance with AUC-ROC. To dissect feature contributions, localized SHAP values will decompose predictions into additive effects, leveraging Shapley's cooperative game theory. For causal inference, Double Machine Learning (DoubleML) will estimate the effect of a binary treatment using orthogonalized residuals through XGBoost fits, satisfying Neyman orthogonality. To accelerate causal structure discovery, Fast Causal Inference (FCI) will construct a partial ancestral graph over efficient conditional independence tests to outpace the PC algorithm, identifying directed edges amid latent confounders. This work aims to fuse predictive accuracy, interpretability, and causal rigor in a unified pipeline. Results will highlight methodological advancements, with validation and testing planned across different timepoints within the comprehensive heart transplant dataset.

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Structure-Aware and Consistency-Preserved Graph Contrastive Learning without Augmentation

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Graph contrastive learning (GCL) has made significant progress in unsupervised graph representation learning. However, most methods rely on manually designed augmentations, which introduce high computational overhead and risk semantic inconsistency—especially when perturbations distort graph structure or corrupt key features. To overcome these issues, we propose SCOPE (Structure-Aware and Consistency-Preserved graph contrastive learning), an augmentation-free framework that exploits intrinsic graph information to define meaningful contrastive objectives.

Specifically, we propose a structure-aware positive sampling strategy using partial absorption scores to select topologically similar nodes as positives, ensuring semantic relevance without artificial noise. Meanwhile, a feature-driven KNN graph serves as an auxiliary view, and consistency between embeddings from the original and KNN graphs is enforced via a cross-view alignment loss. This dual approach removes the need for stochastic augmentations while preserving structural and attribute semantics. We evaluate SCOPE on six benchmark datasets, consistently achieving competitive or superior results compared to other contrastive learning methods. These results highlight the effectiveness of structure-aware sampling and consistency preservation in improving the stability and efficiency of contrastive learning on graphs.

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A High Precision Implementation of Taylor Series Method for the N-body Gravitational Problem

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In this work a high precision implementation of the high-order Taylor series method for the classical N-body gravitational problem is presented. An adaptive step-size strategy is used. Firstly, the formulas for the normalized derivatives in the Taylor series are derived by the automatic differentiation rules and then a simple and fast C-program using the GNU Multiple Precision arithmetic library (GMP library) is realized. The program permits using an arbitrary order of the Taylor series and arbitrary precision. The latter allows us to obtain long-term reliable solutions by applying a verification procedure by comparing the results for gradually increased order and precision. The program is tested on various examples. Comparisons with previously known high-accuracy results are made. The conservation of integrals of motion is also checked and analyzed.

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Application and Development of the Block Maxima Method in Analysis of Silver Price

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The paper presents a review of the Block Maxima approach in the analysis of extreme values in Silver price data over the period from January 3, 1983, to December 31, 2024. The Block Maxima method involves partitioning the dataset into equal time intervals, within which the maximum value of each block is extracted for further statistical analysis. This approach is widely used in extreme value theory to model and predict rare, high-impact events. A mathematical model is proposed for estimating the parameters of the distribution function of the maxima, employing Newton's method. The results provide valuable insights into the extreme behavior of Silver price, offering a robust foundation for risk assessment. The model's efficacy is demonstrated through its application to historical Silver price data, where it successfully captures the tail behavior of the distribution. These findings contribute to the advancement of extreme value modeling in commodity price forecasting and risk management.

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Selection and Verification of an Accurate Mathematical Model of a Proportional Control Valves

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In this paper, experimental investigations of the static flow characteristics of a proportional control valves are presented. To determine the exact mathematical model to describe the actual flow rate through the proportional control valves when the supply pressure, inlet and outlet pressure and fluid path geometry are varied. Verification of the model is performed and an accurate and complete model is selected to serve for subsequent simulation models of pneumatic actuator systems.

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Mathematical Models of the Static Flow response of an Electropneumatic Proportional Control Valves

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This paper presents mathematical models of flow through electropneumatic valve, it is an analysis of existing publications by various authors. The presented models are discussed and described in detail and the conditions for their application to different types of electropneumatic valve designs are presented. Graphical relationships showing the flow through the valves are presented.

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On the Dynamics of Interacting Vortices and Water Waves

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The study of an inviscid incompressible fluid with free boundary is of high theoretical and practical interest, *e.g.*, the evolution of surface waves on the ocean, their approximation by model equations, the dynamics of wave interactions. Usually the surface motion and the motion in the fluid body are considered separately for the sake of simplification of the problem. In many cases the vorticity is the key quantity in the analysis of the fluid motion and then such separation is not possible.

We consider a two-dimensional water-wave problem with a general non-zero vorticity field in a fluid volume with a flat bed and a free surface [1]. The nonlinear equations of motion for the specified surface and volume variables can be written in closed form. These equations arise from Euler's equation and the continuity equation and describe the interaction between the surface and the volume, so that a simple reduction of the model only to the surface variables is not possible.

We illustrate the model with a model of a point vortex and its interaction with the free surface of the fluid. In the small-amplitude long-wave Boussinesq and KdV regimes, we obtain a simplified system of coupled equations for the motion of the vortex and the time evolution of the free surface.

References

[1] D. Ionescu-Kruse, R. Ivanov (2023) Nonlinear two-dimensional water waves with arbitrary vorticity, *Journal of Differential Equations* **368**, 317–349, `https://doi.org/10.1016/j.jde.2023.05.047`.

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On Queueing Delay Distribution in an Analytic Model of a Network Node with Failures

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The transient behavior of queueing systems is of crucial importance in modern communication networks, where nodes may experience random breakdowns affecting the overall system performance. This paper focuses on the transient analysis of the probability distribution of queueing delay in a model of a network node subject to failures in either the transmitter or receiver. The model assumes that incoming messages arrive according to a Poisson process, reflecting a typical random traffic flow. Both the processing time for an individual message and the time of uninterrupted (failure-free) operation are exponentially distributed, capturing the memoryless nature of these processes. In contrast, the repair time following a breakdown is modeled using an arbitrary probability distribution, allowing for a flexible and realistic description of the recovery dynamics.

To analyze the system's behavior, a matrix-based analytical approach is employed. This method enables the derivation of a representation for the conditional probability distribution of the queueing delay at any fixed time instant t in terms of its Laplace transform. The use of the Laplace transform is particularly advantageous in transient analysis, facilitating the handling of complex time-dependent behaviors and the inclusion of general repair time distributions. Additionally, the paper provides several numerical examples that illustrate the theoretical results and highlight how different assumptions about failure recovery impact the queueing delay. These examples demonstrate the applicability of the proposed approach to various practical scenarios, emphasizing its potential use in designing and optimizing resilient network systems.

The results obtained contribute to a deeper understanding of the dynamics of network nodes under non-ideal operational conditions and offer a valuable tool for engineers and researchers dealing with reliability and performance evaluation of communication infrastructures.

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On a Mathematical Model of Virus – Immune System Competition

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Viral infections are widely distributed throughout the world and may cause dangerous diseases. In our paper we present a new mathematical model describing the response of the immune system to viral infection. We study some qualitative properties of the model and perform numerical experiments.

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Amplitude Evolution Equation for a Convective Flow Between Two Vertical Parallel Planes

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We consider weakly nonlinear instability of a convective flow in the region between two parallel vertical planes. Steady flow in the vertical direction is generated due to temperature difference T between two parallel planes. Using the results of linear stability calculations we select the Grashof number (the dimensionless parameter proportional to the temperature difference T) slightly larger than the critical value and apply the method of multiple scales in the neighborhood of the critical point. The amplitude of the eigenfunction of the most unstable mode is assumed to be dependent on an unknown function A of slow longitudinal variable and time. Small Prantl number approximation is used in sequel to construct an asymptotic expansion for the stream function so that the dominant instability mechanism is associated with shear mode. The role of thermal perturbations in the limit of small Prandtl number is negligible so that the heat equation is not considered for weakly nonlinear analysis and the problem reduces to Navier-Stokes equations for incompressible fluid with given convective velocity profile obtained as the solution of the steady Navier-Stokes equations under the Boussinesq approximation. Application of the method of multiple scales to the Navier-Stokes equations leads to the amplitude evolution equation for A. It is shown that the amplitude evolution equation is the complex Ginzburg-Landau equation. The coefficients of

the equation are evaluated in terms of integrals containing linear stability characteristics of the flow and solutions of three boundary value problems for second order linear ordinary differential equations with variable coefficients. Results of numerical calculations are presented.

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Efficiency Analysis of a Transistor Inverter Bridge-Circuit

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The creation of powerful semiconductor devices opens up wide prospects for the development of effective industrial electronics devices on their basis. In particular, the output stages of inverters, converters, induction heating and hardening devices, plasma torches, etc., are built on the basis of bridge and semi-bridge circuits using high-power IGBT transistors and MOSFETs. The use of transistor circuits makes it possible to significantly increase the operating frequency of the inverter, which is of fundamental importance in a number of practical applications.

The paper presents the results of the analysis of equivalent circuits of a bridge and a half-bridge inverter It is shown that the efficiency of the stage is determined by the parameters of the transistors:

- equivalent impedance in the open state;
- the leakage resistance of the transistor in the locked state.

Using the methods of the theory of linear electric circuits, the analysis of the equivalent circuit of the bridge inverter was carried out. Formulas for the dependence of the efficiency of the bridge circuit on the parameters of transistors are obtained, which make it possible to justify the choice of transistors based on the requirements for the implementation of the device. The requirement for the identity of transistor parameters in the bridge circuit is substantiated. An expression for the input

impedance of the bridge circuit is obtained, which allows us to evaluate the energy characteristics of the power supply source of the bridge.

A qualitative analysis of the dynamics of the bridge circuit operation is carried out, and considerations regarding the choice of frequency parameters of transistors are given. A relationship has been established between the value of the “dead time” during the overturning of the bridge, which is necessary for trouble-free operation of the circuit, and the time-domain parameters of transistors. It is shown that the use of MOSFET provides the possibility of increasing the frequency, while IGBT is preferable to use in low-frequency high-current stages. In addition, the choice of inverter transistor types imposes some limitations on the characteristics of the drivers in the input circuits of the stage.

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Some Properties and Applications of the Univariate and Bivariate Non-central Polya-Aeppli Distributions

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A Non-central Polya-Aeppli Stochastic Process (NPAP) with a corresponding Non-central Polya-Aeppli Distribution (NPAD) is considered [1,2]. It is a sum of Polya-Aeppli process and a homogeneous Poisson process. Some nice properties, probability mass function and recursion formulas are also considered. Then by the trivariate reduction method a Bivariate NPAD is introduced [3].

Keywords: Non-central Polya-Aeppli distribution, compound distribution, trivariate reduction method, Bivariate Non-central Polya-Aeppli distribution

References

[1] M.D. Lazarova, L.D. Minkova (2019) Non-Central Polya-Aeppli Process and Ruin Probability, Ann. Acad. Rom. Sci. **11**(2), 312–321.
[2] S.H Ong, P.A. Lee (1979) The Non-Central Negative Binomial Distribution, Biom. J. **21**, 22–26.
[3] H. Papageorgiou (1986) Bivariate ‘short’ distributions, *Communications in Statistics - Theory and Methods* **31**, 871–888.

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A New Spatiotemporal Transformation Different from the Lorentz Transformation

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Being different from calculus operations based on infinitesimal units,infinity can be regarded as an infinite-dimensional infinite quantity that can serves as a basic unit of quantity where a new functional formula is drew. Replacing the functional relationship y and x are one-to-one correspondence in a Cartesian coordinate system in which the x-axis and y-axis are mutually perpendicular straight lines, the relationship between Y and X (X is one-dimension finite quantities and Y is infinite dimensional infinite quantities) is called as a non-correspondence relationship, a non-gradually increasing or decreasing relationship, a nonlinear relationship, or a non-system relationship on this new formula whose length will cover the entire universe if this formula is written out. This formula is a function expression for any curve and can describe a new spatiotemporal transformation that is called the infinite-dimensional transformation. When this infinity is artificially divided into camparable parts in which calculus operation can be applied, the Lorentz transformation where two different inertial frames K system and K' system have the same coordinates but the different speed is substituted by Li transformation where the K and K' are two different states in the one Cartesian coordinate system. A new mass energy relationship can been derived from Li transformation.

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Porous Media-Based Thermal Modeling for Urban Heat Island Mitigation: A Coupled Problem Approach with Optimal Control

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The architectural configuration and urban development of cities significantly influence the mitigation of overheating and the formation of urban heat islands. The integration of natural elements into urban landscapes presents a compelling strategy for alleviating the impacts of urban heat islands, while also enhancing water cycle management and improving urban aesthetics.

This study aims to explore the application of optimal control techniques in the design of green spaces to address the mathematical and environmental challenges posed by urban heat islands. A thermal model based on porous media, specifically the Navier-Stokes-Forschheimer-Fourier system [1,2,3], is developed to evaluate the mitigation of urban heat islands. The study focuses on demonstrating the feasibility and relevance of integrating optimization techniques with the optimal control theory of partial differential equations to enhance thermal comfort in urban environments.

The proposed methodology employs a numerical scheme utilizing the finite element method, the projected spectral gradient algorithm, and the open-source software FreeFem++. The effectiveness and robustness of this approach are evaluated based on the quality of the results derived from a realistic model, demonstrating its potential for realistic model application in urban heat island mitigation strategies.

References

[1] M. Louaked, N. Seloula, S. Sun, and S. Trabelsi (2015) A pseudocompressibility method for the incompressible Brinkman-Forchheimer equations, *Differ. Integral Equ.* **28**, 361–382.
[2] M. Louaked, N. Seloula, S. Trabelsi (2017) Approximation of the unsteady Brinkman-Forchheimer equations by the Pressure Stabilization Method, *Numer. Meth. for Partial Differential Equations* **33**, 1949–1965.
[3] N. Sellila, M. Louaked, W. Mouhali, H. Mechkour (2023) Optimal control strategies for Mitigating urban heat island intensity in porous urban environments, *Mathematics* **11**(23), 4737.

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Low-frequency Vibration Attenuation in Metamaterial Beams with Embedded Inerters: Modeling and Dynamic Analysis

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Effective suppression of low-frequency vibrations is key to ensuring the stability of engineering structures and mechanical systems. However, achieving such suppression remains a significant challenge, as traditional structures exhibit inherent limitations in mitigating low-frequency vibrations. To address this issue, this study investigates the dynamic characteristics and band gap tunability of a metamaterial beam system based on a coupled 2-DOF vibration model with embedded inerters. Firstly, the motion equations of a simplified 2-DOF model with embedded inerters are formulated, the transfer function is obtained via Laplace transform. Second, the Routh-Hurwitz stability criterion is employed to derive the necessary and sufficient conditions under which the proposed oscillator remains stable. Next, building on this foundational model, a metamaterial beam structure is proposed and analyzed. Then, the frequency response and dispersion relation of the flexural wave are obtained using the harmonic balance method. Finally, the analytical expressions for the band gap boundaries are derived, revealing the distinct roles of the inerter parameters in shaping the attenuation characteristics. These findings offer valuable insights for the design and optimization of metamaterials with enhanced vibration attenuation capabilities.

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Stock Prices Prediction Using Hybrid Convolutional Neural Network/Long Short-Term Memory Network Model

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Stock market prediction remains a challenging task due to the volatile and nonlinear nature of financial time series. In this study, we propose a hybrid deep learning architecture combining Convolutional Neural Networks (CNN) and Long Short-Term Memory (LSTM) networks for predicting stock closing prices. The model leverages both spatial feature extraction and temporal sequence modeling to enhance predictive performance. Historical daily stock data for five major technology companies (AAPL, MSFT, AMZN, GOOGL, and NVDA) from January 2015 to July 2024 was collected using the Yahoo Finance API. To enrich the input features, we incorporated technical indicators including the 20-day Simple Moving Average (SMA), the 14-day Relative Strength Index (RSI), and the Moving Average Convergence Divergence (MACD). The dataset was normalized and transformed into time-series sequences suitable for deep learning input. We compare the performance of the CNN-LSTM model trained with and without these technical indicators to assess their contribution to forecasting accuracy. Preliminary results suggest that the integration of domain-specific indicators significantly improves the model's ability to capture price trends and reduce overfitting. This approach highlights the effectiveness of hybrid architectures in financial forecasting and the value of embedding technical analysis into deep learning frameworks.

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Stochastic Monte Carlo–FEM Prediction of Electrical Conductivity in Carbon-Nanoparticle/Polymer Composites for Green Technologies

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A hybrid stochastic framework is presented for forecasting the bulk electrical conductivity of PVDF composites loaded with graphene nanoplatelets (GNP) and multi-walled carbon nanotubes (MWCNT). The method couples a three-dimensional Monte Carlo algorithm—generating thousands of statistically independent microstructures with random particle positions, orientations and aspect ratios—with finite-element solution of the steady-state diffusion (Laplace) equation to compute ensemble-averaged current pathways. Sensitivity analysis highlights that increasing the nanotube aspect ratio halves the percolation threshold, while graphene sheets stabilise the network without requiring a fixed mass ratio. The proposed Monte Carlo–FEM approach offers a fast, physics-based tool for designing lightweight, highly conductive composites suitable for flexible current collectors, electromagnetic shielding and other green-technology components. Future work will extend the model to include non-linear tunnelling effects and temperature dependence to enhance predictive accuracy under service conditions.

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A Model for Measuring the Economic Effects of Implementing Artificial Intelligence Technologies in Business Practices

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Thanks to recent advancements in Artificial Intelligence (AI) and machine learning, business is transforming profoundly. We are entering a new era for organizations and individuals, one in which the fundamental principles of business management are being completely redefined. To explore the magnitude of this transformation, this study develops a model for measuring the economic effects of AI integration in business. The main goals of the research are:

To track the process of AI implementat-ion in businesses and examine the resulting impacts.

To investigate how artificial intelligence and machine learning enhance added value through automation, process optimisation, and improved customer service.

To assess how the adoption of AI technologies reshapes job structures, workforce skill requirements, and strategies for reskilling and upskilling.

To identify and validate key performance indicators (KPIs) that measure economic benefits of AI, such as cost reduction and revenue growth.

To analyse the influence of AI and machine learning on business competitiveness at both national and international levels.

To evaluate how AI technologies increase customer satisfaction and loyalty, ultimately generating financial gains for businesses.

The study collects data from three companies—two software firms and one international corporation—to assess the impact of AI across diverse industries. Data collection includes both qualitative and quantitative methods: interviews with business leaders, employee perception surveys, and process analysis. Financial reports, productivity metrics, and AI usage statistics are also examined. Econometric models are applied to capture linear or simple relationships and determine how increasing investments in AI affect profitability. Machine learning techniques are used to identify more complex patterns that cannot be captured through linear models, particularly when AI outcomes depend on multiple variables such as team size, business type, and operational scale. Additionally, a qualitative analysis is conducted through feedback from employees and customers. This includes calculating the financial and operational effects of AI in areas such as revenue growth, workflow optimization, and enhanced market competitiveness. A visualization tool is developed to present the collected data and model outcomes, making insights easily accessible and actionable. This tool is tested in a real-world environment to evaluate its practical applicability. In conclusion, businesses are already adopting AI-driven systems, from machine learning to computer vision and deep learning. Some companies are taking a cautious approach, seeing only modest productivity gains in the short term. However, these gains may plateau over time. Others, by embracing innovation, achieve game-changing performance improvements that redefine their competitive advantage.

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Advancing Predictive Maintenance Modeling for Renewable Energy Systems

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Predictive maintenance plays a pivotal role in extending the operational lifespan of renewable energy infrastructure while minimizing unplanned downtime and maximizing energy efficiency. In this study, we investigate the application of traditional time-series modeling techniques to support fault detection and maintenance forecasting in wind turbine systems. Using a high-resolution SCADA dataset comprising 89 years of multivariate sensor data from 36 turbines across three wind farms, we focus on statistical models that are interpretable, robust, and computationally efficient.

Our methodology involves preprocessing turbine data streams—including wind speed, rotor speed, power output, and bearing temperatures—and applying classical time-series models such as ARIMA (AutoRegressive Integrated Moving Average), SARIMA (Seasonal ARIMA), and Vector AutoRegression (VAR) to capture temporal dependencies and seasonal behaviors.

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Data-centric Reliability Modeling for Energy Assets

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In modern energy systems, the reliable operation of critical assets—from wind turbines and photovoltaic arrays to transformers and gas turbines—underpins both economic performance and grid stability. Traditional reliability models often rely on limited failure data and expert judgment, constraining their accuracy and adaptability. This paper presents a data-centric framework for reliability modeling of energy assets that leverages heterogeneous operational and condition-monitoring datasets to derive dynamic, asset-specific failure predictions. We integrate advanced statistical techniques with machine-learning methods (such as gradient-boosted trees and

deep autoencoders) to capture both time-dependent degradation trends and complex, multivariate interactions among sensor signals. Our approach is demonstrated on two case studies: predictive maintenance of offshore wind-turbine gearboxes and lifecycle prognosis for high-voltage transformers. By shifting the focus from static, rule-based reliability assessments to adaptive, data-driven prognostics, this work offers a scalable pathway for enhancing the resilience and cost-effectiveness of energy infrastructures.

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Dynamic Characterization of Aircraft Propellers under Erosion Cracks

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In severe wind-sand environments such as plateaus and deserts, the propeller blades of large turboprop transport aircraft are continuously impacted by high-speed sand particles, inducing erosion wear that causes material loss and initiates micro-cracks. Such damage reduces the blade cross-sectional stiffness, significantly affecting its vibration characteristics. Under cyclic loading, cracks may propagate and even lead to fracture, posing a critical threat to flight safety. This study investigates the dynamic response of a cracked rotating composite laminated beam under cyclic loads. The edge crack of the blade is modeled as a dimensionless spring in the rotating state. Based on Hamilton's principle, the governing equations for the cracked rotating laminated beam are established. The harmonic balance method is employed to solve the natural frequencies (eigenvalue problem), focusing on analyzing the influence of crack depth, crack location, rotational speed, and installation angle on free vibration characteristics. Meanwhile, the Runge-Kutta numerical integration method is used to study the steady-state response, exploring the mechanism of crack parameters on vibration response. The results show that increasing crack depth, closer crack location to the blade root, higher rotational speed, and larger installation angle significantly affect the blade vibration characteristics. This research provides essential theoretical support and engineering references for the design optimization and preventive maintenance of turboprop propeller blades in harsh wind-sand environments.

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Advanced Stochastic Techniques for Sensitivity Analysis in Large-Scale Air Quality Modeling

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This paper presents an improved approach to multidimensional sensitivity analysis through the application of advanced stochastic simulation techniques, aimed at modeling the spread of air pollutants across vast geographical areas. The study utilizes the Unified Danish Eulerian Model (UNI-DEM), a well-established framework in atmospheric science, to investigate the long-range transport of pollutants and their environmental impact. Emphasis is placed on refining traditional Monte Carlo and quasi-Monte Carlo algorithms by incorporating sophisticated lattice-based and digital low-discrepancy sequences. These enhancements are designed to improve the precision and efficiency of numerical integration tasks fundamental to environmental modeling. Our research specifically analyzes the sensitivity of UNI-DEM outputs to variations in anthropogenic emissions and selected chemical reaction parameters. To this end, we compute global Sobol sensitivity indices, which quantify the influence of different inputs on pollutant concentration levels. The simulations encompass various European urban regions, capturing a range of topographical and climatic conditions. The results demonstrate the practical potential of modern stochastic methods in high-dimensional environmental simulations and their role in guiding effective air quality policy.

Acknowledgement. Venelin Todorov is supported by BNSF under Project KP-06- N62/6.

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Application of LiDAR and Photogrammetric Technologies for Imaging, Mapping and 3D Modeling of Underground Karst Objects – Results from Experiments in the Cave Svirchovitsa near Karlukovo Village, NW Bulgaria

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LiDAR (Light Detection and Ranging) is a technology for non-contact distance measurement with high accuracy and speed. It uses laser pulses to measure the distances from a given point (station) to all objects visible from it and to create three-dimensional maps of surfaces. LiDAR systems send laser light pulses and measure the time it takes for these pulses to reflect back, which allows the distances to objects to be determined accurately.

In this article we demonstrate the first steps in application of LiDAR and complementary technologies for imaging, mapping, and 3D modeling of a particular karst object – Svirchovitsa cave in Karlukovo karst region. We discuss the principles of operation, technological integration, field data acquisition methods, and the processing methods used to obtain 3D models from the raw images (including sensor data and markers). By this case study we'll demonstrate the applicability and practical benefits of these novel technologies in speleological research, risk assessment, as well as for conservation and tourist purposes. LiDAR profile element of a cave gallery in Svirchovitsa cave in Kamenopole-Karlukovo karst region, NW Bulgaria

By leveraging these advanced tools, researchers, constructors and visitors can gain deep insight into the geometry, structure, and evolution of karst systems. In perspective, this research would open new possibilities for exploration, monitoring, and sustainable management of these hidden, often fragile and sometimes dangerous environments.

Acknowledgements. This work is supported by the project "Integrated approach to creating digital twins of archaeological immovable monuments using innovative technologies," contract KP-06-N82/1 with the Bulgarian National Science Fund. We acknowledge also the access provided to the E-Infrastructure of the Laboratory for 3D Digitization and Microstructural Analysis at the Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, Grant No BG05M20-P001-1.001-0003

Keywords: 3D model; LiDAR; photogrammetry; cave; karst; speleology

References

[1] G. Vasilev, H. Klecherova, G. Evtimov, M. Raykovska, V. Petrova, I. Georgiev, Comprehensive digitization of the "Kazlacha" circular enclosure complex: an adapti-

ve synergistic approach, *Studies in Computational Intelligence* (Springer Verlag, ISSN: 1860-949X) (to appear).
[2] S. Antinozzi, A. di Filippo, D. Musmeci (2022) "Immersive photographic environments as interactive repositories for preservation, data collection and dissemination of cultural assets," in: *Heritage*, Vol. **5**(3), pp. 1659–1675.
[3] N. Prahov, B. Prodanov, K. Dimitrov, L. Dimitrov, K. Velkovsky (2020) "Application of Aerial photogrammetry in the study of the underwater archeological heritage of Nessebar," in: *20th International Multidisciplinary Scientific GeoConference Proceedings SGEM 2020*, pp. 175–182.

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An Extension of the Definition on the Compositions of Generalized Functions and Powers of Some Singular Generalized Functions Defined as the N-limit of Regular Sequences

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The compositions of generalized functions appear in many physics problems. For instance the symbol δ^2 often appears in quantum mechanics which leads to reasonable results and further the symbol δ^{-1} appeared in cosmological models [1,2] in which the following distributional identities

$$\big[f(\omega) + C\delta(\omega)\big]^{-1} = f^{-1}(\omega), \qquad \frac{d}{d\omega}\big[f(\omega) + C\delta(\omega)\big]^{-1} = \frac{d}{d\omega} f^{-1}(\omega)$$

were used. In the theory of generalized functions, there is no reasonable way of introducing neither the square δ^2 nor δ^{-1}.

Gelfand and Shilov give the definition of the composition $\delta(f(x))$ for an infinitely differentiable function $f(x)$ having any number of simple roots. The aim of this talk is to extend their definition, in fact, give the definition of the composition $\delta(f(x))$ for an infinitely differentiable function having any number of multiple roots by using the method of the discarding of unwanted infinite quantities from asymptotic expansions. Further we define the k-th powers of this composition which will be the N-limit of the regular sequence $\delta_n^k(f(x))$ for positive integers. By using the method of the discarding of unwanted infinite quantities from the divergent integral via taking the N-limit, we define the s-th powers of the Heaviside function $H(x)$ in the distributional sense for negative integers.

Acknowledgement. This work is supported by the Scientific Research Projects Coordination Unit of Hacettepe University – Proj. Num. FHD2024-21165.

Key words and phrases: Divergent integral, regular sequence, Hadamard's finite part, Dirac-delta function, composition of distribution, neutrix limit, Faá di Bruno's formulae, Bell polynomials.

2020 Mathematics Subject Classification: 41A30, 46F10, 40A10, 11B83.

References

[1] A.Y. Kamenshchik (2013) Quantum cosmology and late time singularities, *Classical and Quantum Gravity* **30**, 173001, 44pp, `doi:10.1088/0264-9381/30/17/173001`.
[2] Z. Keresztes, A.L. Gergely, A.Y. Kamenshchik (2012) Paradox of soft singularity crossing and its resolution by distributional cosmological quantities, *Physical Review D* **86**, 063522 12pp `https://doi.org/10.1103/PhysRevD.86.063522`.

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Non-self-adjoint Sixth-order Eigenvalue Problems Arising from Clamped Elastic Thin Films on Closed Domains

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Sixth-order boundary value problems (BVPs) arise in thin-film flows with a surface that has elastic bending resistance. We consider the case in which the elastic interface is clamped at the lateral walls of a closed trough and thus encloses a finite amount of fluid. For a slender film undergoing infinitesimal deformations, the displacement of the elastic surface from its initial equilibrium position obeys a sixth-order (in space) initial boundary value problem (IBVP). To solve this IBVP, we construct a set of odd and even eigenfunctions that intrinsically satisfy the boundary conditions (BCs) of the original IBVP. These eigenfunctions are the solutions of a non-self-adjoint sixth-order eigenvalue problem (EVP). To use the eigenfunctions for series expansions, we also solve the adjoint EVP, leading to another set of even and odd eigenfunctions, which are orthogonal to the original set (biorthogonal). The eigenvalues of the adjoint EVP are the same as those of the

original EVP, and we find accurate asymptotic formulas for them. Next, employing the biorthogonal sets of eigenfunctions, a Petrov-Galerkin spectral method for sixth-order problems is proposed, which can also handle lower-order terms in the IBVP (such as second-order in space terms due to gravity). The proposed method is tested on two model sixth-order BVPs, which admit exact solutions. We explicitly derive all the necessary formulas for expanding the quantities that appear in the model problems into the set of eigenfunctions. For both model problems, we find that the approximate Petrov-Galerkin spectral solution is in excellent agreement with the exact solution. The convergence rate of the spectral series is rapid, exceeding the expected sixth-order algebraic rate (*i.e.*, $|a_N| = O(N^{-6})$, where a_N are the coefficients of the expansion, and N is the number of terms in the spectral series).

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Causal Relationship Between Bank Lending and Its Determinants: Evidence from Bulgarian Commercial Banks (2007-2024)

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This study analyses the causal relationship between bank lending and its determinants in a panel of 21 privately owned Bulgarian commercial banks, over the period 2007Q1–2024Q4 using quarterly data. Our I(1) data suggests that bank-specific and macroeconomic factors Granger cause and homogeneously cause bank lending, using Panel Granger and the Dumitrescu-Hurlin panel Granger causality tests. The causality analysis is combined together with cointegration testing procedures, using the Kao residual cointegration test, based on the Engle-Granger approach. Through cointegration equations we identify the long-term relationships between bank lending and its determinants. A panel Fully Modified Ordinary Least Squares

(FMOLS) estimator is used to quantify the long-run elasticities. Bank specific factors, such as interest rates on bank liabilities, bank concentration, market share, loans to assets ratio, loans to deposits ratio, deposit and assets values, together with macroeconomic variables, such as house prices, inflation, GDP have long-term impact on bank lending.

Acknowledgement. This work was financially supported by the UNWE Research Programme (Research Grant No. 29/2025/A)

Keywords: Panel regression, Granger causality, Dumitrescu-Hurlin, panel cointegration, FMOLS, bank lending, credit growth, macro-financial linkages, Market concentration

JEL Classification: C33; E44; G21; E51

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Capital Asset Pricing Model in Insurance Risk Theory

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One of the latest trends in the insurance business is the development of investment strategies to increase capital. The use of profit from financial investments can significantly enhance the assessment of insurance risk. In this paper, a combined model between the Capital Asset Pricing Model and the classical risk theory in the form of the Cramer-Lundberg model is considered.

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Bayesian Approach for Bulgarian Insurance Company Classification According the Policy Premiums

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The Bayesian approach is very useful and preferred due to it's easy understanding and logic. It finds place in many models in insurance mathematics. In the current paper is considered the question of classification of the insurance company on the Bulgarian market according to the registered premium incomes. The information for the premium values could be approximately convert to information about the number of policies. Furthermore, for the distribution of the number of policies could be applied Bayesian formula for construct different categories, which could be useful for the insurance business and for its clients.

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Equations in Infinitely Many Derivatives and the Zeta Function

E. Reyes

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I will present some contemporary work on the analysis of equations in infinitely many derivatives such as

$$\Delta \exp(-c\Delta)\phi = U(x, \phi) \ ,$$

in which Δ is the Laplace operator, or

$$\zeta(\partial_t)\phi = g(t)$$

in which ζ is the Riemann zeta-function. These equations appear in string theory and cosmology, and a challenging endeavour is to develop mathematical frameworks in which they can be studied rigorously. In this talk I will summarize some of the work carried out along these lines.

This research is mainly a collaboration with Humberto Prado (Santiago, Chile), Przemyslaw Gorka (Warsaw, Poland) and Alan Chavez (Trujillo, Peru).

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Simulations and Analysis of Mesoscale Cyclones in the Black Sea Area Based on Satellite Data

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Numerical simulation on the base of a low-parameter nonlinear model of tropical cyclogenesis applied to mesoscale storms in the Black Sea region was performed. The numerical experiments were conducted both for the Mediterranean storm Daniel and for the mesoscale storm appeared in southwest Black Sea area. Satellite data in the form of radar and visible spectral images from multiple sources for these storms were collected. The topology, geometrical dimensions and trace of the storm Daniel was analyzed. Special attention was paid to the mesoscale storm developing over the sea and over land in the southwestern part of the Black Sea. The temporal dynamics of precipitation density based on satellite radar images was studied. Differences between predictions of two main global models and measured radar densities were discussed. Infrastructure and environmental damage images caused by this mesoscale storm are provided. The possibility of implementing an accurate forecast of such events in this region was assessed.

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Non-stationary Wave Dynamics in Compound Structures: Mathematical Modeling and Analysis

O. Korotunova, G. Shyshkanova, T. Zaytseva, N. Levchenko

Humboldt-University in Berlin, Germany

Structures comprising elements with markedly distinct physical and mechanical properties are ubiquitously employed in engineering applications, encompassing domains such as mechanical systems design and structural engineering. The paper

considers the motion of an elastic half-strip interacting unilaterally with an absolutely rigid foundation, caused by a longitudinal edge mechanical fast impulse load. Dynamic equations of elasticity theory are used to describe the motion of the half-strip. The method of spatial characteristics is used to model the wave dynamics and dynamic failure of a piecewise homogeneous structure. This method allows to take into account variable conditions at the contact surface of the half-strip and the foundation, and consider delamination processes. The connection of the half-strip to the foundation is modeled by a nonlinear strength function of normal and tangential stresses, taking into account the mechanism of the connection, including its separation. The propagation of elastic waves caused by a non-stationary mechanical action is studied. The calculation results are analyzed to identify the areas of the contact surface that are most susceptible to damage. In order to study the shape of the incident, reflected and transmitted load pulse at the contact boundary, distributions of the values of the velocity components and stresses for different moments in time were obtained. Applications such as advanced non-destructive testing, energy-efficient designs, and responsive material systems benefit from wave research techniques. The focus on propagation of waves in compound structures could further bridge theoretical advancements with practical implementations, enhancing the structural reliability and functionality of dynamic systems. The proposed method of direct numerical solution allows us to give practical recommendations for the identification of zones of emerging local stress concentrations at the mating points.

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A Multicriteria Optimization Approach for Investment Project Funding

I. Georgiev, Sl. Georgiev, B. Chakarov

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M. Lazarova

Technical University of Sofia, Bulgaria

Investment projects frequently undergo evaluation based on diverse criteria, including economic, social, environmental, and additional relevant factors. Each project is assigned points reflecting its performance in these criteria. Due to limited budgets, not all qualified projects can be fully funded. This creates a challenging decision-making situation, where funding organizations need to select the best projects while maximizing both the number of ap-proved projects and their overall quality. Additionally, practical funding de-cisions often result in partial rather than full financial support, requiring frac-tional resource allocation. This paper proposes a multicriteria mixed-integer optimization model ex-plicitly designed to allocate limited financial resources among competing in-vestment projects by maximizing both the number of approved projects and their cumulative quality scores. The proposed approach allows partial fund-ing, thereby enhancing flexibility and ensuring efficient utilization of availa-ble resources. The applicability and benefits of the proposed model are demonstrated through illustrative examples.

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Panel Data Econometric Techniques for Modeling Bank Loan Supply in Bulgaria: Evidence from 21 Commercial Banks (2007-2024)

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University of National and World Economy, Sofia, Bulgaria

This article studies factors determining real loan growth in the Bulgarian banking sector, using quarterly data from 21 commercial banks over the period Q1.2007 to Q4.2024. In working with I(0) and I(1) data we apply various panel methods for identifying short-term and long-term factors for bank loans' growth, including Panel differenced OLS, Cointegration, Random Effects (RE), System GMM, and a Pooled Mean Group (PMG) estimator (Panel ARDL) framework. This multi-model approach allows us to address unobserved heterogeneity, potential endogeneity, and the role of lagged effects in bank lending behavior of banks. Our findings suggest that higher capital adequacy ratios negatively affect loan dynamics, highlighting a potential trade-off between regulatory requirements and banks' lending activity. Banks with growing market share and higher loan-to-deposit ratios tend to expand their credit faster. While differenced OLS and RE models suggest a positive link between profitability and lending, the GMM results indicate that banks with higher profitability may reduce lending in the short run. The ARDL model confirms a long-run equilibrium relationship between lending and its key bank-specific and macroeconomic determinants. The error correction term is negative and suggests that around 11% of any short-run deviation from equilibrium is corrected each quarter.owledgement

Acknowledgement. This work was financially supported by the UNWE Research Programme (Research Grant No. 29/2025/A)

JEL Classification: C33 ; C32 ; E51 ; G21

Keywords: Bank supply; Bank lending; Panel regression; GMM, Pooled Mean Group (PMG); Panel ARDL; Bulgaria; Random Effects OLS; Macroeconomic factors

→ ∞ ⋄ ∞ ←

Brain Pattern Analysis For Epileptic Activity Using Novel EEG Source Localization Methods

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Brain electric activities are fundamental for brain functions. It analysis using Electroencephalogram (EEG) analysis plays an important role in brain research and medicine. The well-known Hodgkin-Huxley theory for neurons laid a foundation for computational neuroscience. However, understanding activities in the whole brain remains a focus of active research for this very complex system. Full brain Electroencephalography (EEG) and its source localization is a brain imaging modality based on multi-channel EEG signals. It measures the brain field potential fluctuations on the entire scalp for a period of time, and then we can compute the electric current density inside the brain by solving an inverse problem for an electric field equation on the 3-D brain finite element model. In this talk, we introduce computational methods for the EEG imaging problems, their validations through experimental data, and discuss its applications. One application is in identifying brain activity abnormalities and the sequence of excitation in brain anatomic areas during seizures of infant patients with Glucose Transporter Deficiency Syndrome. Our research shows the EEG data sets can be used to glean into the inner working of brain normal and pathological functions in specific brain areas using data analytic algorithms to solve inverse problems for field potential equations.

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Advanced Monte Carlo Techniques for High Dimensional Fredholm and Volterra Integral Equations

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S. Apostolov

St. Kliment Ohridski University of Sofia, Bulgaria

This paper presents advanced Monte Carlo methods for the numerical solution of high-dimensional Fredholm and Volterra integral equations, which frequently arise in mathematical physics, finance, and engineering. Traditional deterministic techniques often suffer from the curse of dimensionality, making them computationally infeasible in complex applications. The results highlight the potential of these advanced methods to significantly improve the efficiency and applicability of integral equation solvers in high-dimensional settings.

Acknowledgement. The work is supported by BNSF under Project KP-06-N62/6 Machine learning through physics-informed neural networks.

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Bioeconophysical Science of Microeconomical Equilibrium of Stocks

M. Petrov, S. Tranev

Prof. Assen Zlatarov State University, Burgas, Bulgaria

This interdisciplinary study presents an integrative platform at the intersection of thermodynamics, econophysics, pharmaceutical sciences, and human health. It investigates the bioeconophysical equilibrium of the human system and its relation to microeconomic behavior in pharmaceutical markets. Specifically, the study explores how the molar mass of medicinal substances affects key pharmacokinetic parameters such as half-life and drugreceptor interactions, interpreted through thermodynamic principles.

An econophysical model, analogous to the ideal gas law, is introduced to describe microeconomic systems of stocks. The model proposes a relationship between molar mass and drug half-life, demonstrating that compounds with higher molecular weights typically exhibit longer half-lives due to decreased membrane permeability. Additionally, a bioeconophysical equilibrium equation is derived, linking drug prices, molar masses, and receptor activation energies. This equation is empirically validat-ed using real pharmaceutical data.

The research also proposes a novel framework for determining optimal drug administration intervals based on molar mass, supported by both theoretical derivations and empirical evidence. The conclusions emphasize the integration of biophysical and econophysical concepts, offering novel insights into drug development strategies and economic dynamics within pharmaceutical systems. This work highlights the unifying potential of bioeconophysics in addressing complex biological and economic phenomena.

Key words: Bioeconophysics, Drug-receptor interactions, Half-life molar mass, Thermodynamics

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Bioeconophysical Integrated Analysis of the Complex System Biosphere-Technosphere-Nosphere and the Role of Entropy and Albedo in System Sustainability

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V. Todorov

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This interdisciplinary study integrates concepts from biology, economics, and physics to analyze the dynamics of the complex system comprising the Biosphere–Technosphere–Noosphere (BTN). The analysis is based on the thermodynamic parameter entropy, which enables the assessment of the state of each subsystem within the BTN framework. Using thermodynamic principles, the study quantifies the impact of anthropogenic pollutants on the Biosphere and how resulting entropy changes propagate into the Technosphere and Noosphere.

A novel relationship is derived between the accumulated anthropogenic heat in the Biosphere and albedo—a physical indicator reflecting the physicochemical properties of environmental materials. Changes in albedo due to pollution influence entropy levels, biological productivity (such as photosynthesis), and associated economic costs. High albedo values correlate with entropy dissipation, reduced photosynthetic activity, and increased vulnerability to climate extremes including droughts, floods, hurricanes, and wildfires. These events lead to chaotic entropy surges and economic losses.

The findings highlight albedo as not merely a physical parameter, but as a critical connector between thermodynamic disorder (entropy), ecological stability, and economic sustainability.

Key words: Albedo, Bioeconophysics, Biosphere, Economic efficiency, Entropy, Sustainability, Technosphere, Thermodynamics

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Using Machine Learning Techniques in Robotics

Y. Vasileva

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Machine learning is rapidly transforming the field of robotics by enabling systems to perceive, learn, adapt, and make intelligent decisions in complex and dynamic environments. This paper provides an overview of how machine learning techniques are applied across key areas in robotics, including perception, motion planning, control, localization, and human-robot interaction. We explore supervised, unsupervised, and reinforcement learning methods, highlighting their role in tasks such as object recognition, trajectory prediction, behavior cloning, and adaptive control.

Real-world case studies demonstrate the application of machine learning in autonomous navigation, robotic manipulation, and predictive maintenance, using both simulated and physical platforms. Particular emphasis is placed on time-series learning for motion analysis and sensor fusion.

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The Synergy of Mathematics, Computer Science and Generative Artificial Intelligence

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Complex mathematical concepts, such as probability, statistics, linear algebra, and optimization are the essence of Generative Artificial Intelligence (GenAI) are. These mathematical frameworks provide the theoretical foundation for developing generative models that can learn from large data sets and generate new data that is similar to the original one. Computer science plays a crucial role in translating these mathematical concepts into practical algorithms and software tools. Neural networks, deep learning algorithms, high-performance computing platforms are crucial for training and deploying generative models. The synergy between mathematics and computer science is present in various applications of GenAI. Generative adversarial networks (GANs) and variational autoencoders (VAEs) are typical

examples of powerful models that can generate realistic images and videos. Advanced language models provide completely new forms of processing, enabling text, image, and video generation.

Despite the enormous potential of GenAI, it is essential to discuss the ethical and social implications associated with its development and use. Risks such as the creation of fake news and deepfakes, as well as biases in data and algorithms, need to be mitigated through the development of ethical guidelines, regulations, and transparent models. The synergy between mathematics, computer science and GenAI is a powerful way that is transforming various industries and reshaping the future of technology.

The paper explores the complex synergy between mathematics, computer science, and GenAI, highlighting how their joint interaction drives innovation and transforms diverse fields. The paper aim is to uncover the complex connections between these three disciplines and demonstrate how their unification enables the creation of powerful generative models that can generate new and meaningful data.

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Global Quantitative Stability of Wave Equations with Strong and Weak Dampings

Runzhang Xu

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In this talk, we are concerned with the description of global quantitative stability of wave equations with linear strong damping and linear or nonlinear weak damping. By giving some energy decay estimates, we obtain several conclusions about the continuous dependence of the global solution on the initial data and the coefficients of the strong damping term and linear or nonlinear weak damping term. This work also establishes a new idea to use the dissipative effect to obtain the better continuous dependence conclusions, which also reflect the dissipative properties of the solution.

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Dynamic Analysis of Aircraft Engine-Pylon Coupled System

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The pylon structure is a critical component linking the engine and wing in large civil aircraft, responsible for securing the engine and transferring its loads to generate propulsion. Simultaneously, it also transmits engine-induced vibrations to the airframe, posing risks to flight safety. Traditional methods such as analytical modeling and finite element analysis (FEA) have limitations in accurately capturing the vibration behavior of the pylon. Analytical models often oversimplify the structure, while FEA models, though detailed, are computationally intensive. To address these limitations, this thesis proposes a concentrated mass-bent beam model of the pylon to balance modeling accuracy and efficiency.Firstly, a dynamic model of the pylon is developed using the transfer matrix method, and its parameters are refined through sensitivity analysis based on FEA results. The model's low-order natural frequencies are compared with those from FEA to validate its effectiveness. Secondly, this pylon model is integrated with an engine excitation model to form an engine-pylon coupled dynamic system. Vibration characteristics of the system are analyzed under engine operating conditions such as take-off, cruise, and descent. Vibration envelopes and time-domain responses of key mass units are obtained for each case. Third, a scaled pylon model is designed based on similarity theory, and the corresponding experimental setup is constructed. Comparative analysis between the experimental data and theoretical results confirms the validity of the proposed model.This work establishes a practical and accurate approach to studying the vibration behavior of the engine-pylon system. The proposed modeling strategy enhances vibration prediction and control, offering valuable insights for improving flight safety and structural performance in civil aviation.

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Hypergraph Mamba-based Dual-channel Denoising Network for Session-based Recommendation

Yafang Li, Miao Wang, Baokai Zu, Caiyan Jia
Beijing University of Technology, China

In the modern internet era, characterized by the widespread proliferation of smart devices, the rapid generation and dissemination of information have led

to the problem of "information overload." Recommender systems, as a decision-making support technology in the field of artificial intelligence, have emerged as a solution. Session-based recommendation (SBR) algorithms have attracted considerable attention in practical applications. However, existing SBR models exhibit limitations in extracting temporal and semantic relationships within sessions and suffer from redundancy in utilizing global information. To address these issues and improve recommendation quality, we propose the Hypergraph Mamba-based Dual-channel Denoising Network (HM-DDN). HM-DDN models sequence features at both the global and session levels. By introducing transition hyperedges and context hyperedges into the hypergraph structure, it enables the accurate modeling of both temporal dependency relationships between items in sessions and the local semantic information of item sequences, thereby achieving high-order representation of item transition relationships. Based on high-order information embedding, the model uses the Mamba mechanism to efficiently extract long-term dependency features from session sequences. In the global modeling, degree-sensitive pruning is introduced for information denoising, which enhances the model's ability to focus on critical information by removing low-confidence connections from the global graph. We validate the model on two public datasets, and the experimental results demonstrate that HM-DDN significantly outperforms existing methods in session-based recommendation tasks.

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A Privacy-preserving Scheme for External Security in Blockchain-based Federated Learning

Hongyuan Wang, Baokai Zu, Jianqiang Li, Jingbang Wu, Wanting Zhu
Beijing University of Technology, China

Federated learning (FL) is a new paradigm for achieving data security, as it enables multiple participants to collaboratively train machine learning models without data being exported locally. The decentralized characteristic of blockchain enables FL to avoid privacy leakage and other security issues caused by central single point failures. However, there are external adversaries who can easily obtain some data privacy by monitoring system data, intercepting transmission information, and analyzing the behavioral characteristics of data, which easily lead to system privacy leakage and cause significant losses to individuals or businesses.

Aiming at the above issue, this paper proposed a privacy-preserving scheme for external security in blockchain-based federated learning (ES-BFL) based on differential privacy, signature, encryption and data obfuscation technologies. The

ES-BFL scheme adopts a combination of differential privacy and data obfuscation to ensure the privacy and security of local model parameters, while using a combination of encryption and signature to ensure the privacy and security of global model parameters, thereby ensuring the external privacy security of the entire BFL system. Finally, a security analysis is given under the cryptographic standard model and the experiments and data analysis are conducted in a simulated environment. Preliminary experimental data shows that the ES-BFL scheme can efficiently ensure privacy security without affecting the accuracy of federated training.

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Generative Adversarial Network with Vision Transformer for Pollen Grain Microscope Images Classification

Baokai Zu, Zhengrui Yang, Tong Cao, Hongyuan Wang, Xu Li

Beijing University of Technology, China

Pollen allergy has emerged as a significant seasonal health concern, often manifesting as an epidemic with high incidence rates in urban environments. Accurate classification and identification of airborne allergenic pollen types are of critical importance for enhancing public health measures, improving urban livability, and supporting early-warning systems for allergy-prone populations. Traditional methods of pollen classification rely heavily on manual identification and conventional image processing techniques, which are often time-consuming and prone to error. With recent advancements in deep learning, particularly in the field of computer vision, Vision Transformers (ViTs) have demonstrated competitive performance over tradi-tional Convolutional Neural Networks (CNNs) in various image recognit-ion tasks. In this study, we propose the use of a generative model, namely the Vision Transformer-based Generative Adversarial Network (ViTGAN), for the classification of allergenic pollen images. ViTGAN combines the powerful feature extraction capabilities of ViTs with the data augmentation and synthesis strengths of GANs, offering a robust framework for pollen image classification. Building upon the generative advantages of ViTGAN, we apply this model to a curated dataset of allergenic pollen collected from the Beijing metropolitan area. Experimental results validate the effectiveness of the proposed approach, demonstrating that ViTGAN achieves superior classification accuracy compared to conventional CNN-based GAN models. Our findings suggest that ViTGAN holds strong potential as a practical tool for pollen classification.

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Dual-Path Network-Based Brain Tumor CT Image Anomaly Detection System

Liyi Ying, Shaotao Zhu, Yupeng Liu
Beijing University of Technology, China

Brain tumors are diseases that pose a serious threat to human health in the current medical field. Early diagnosis and treatment largely determine the survival rate of patients. Currently, computed tomography (CT) is the most commonly used imaging method for detecting brain tumors. However, the screening of CT images mainly relies on manual inspection, which has issues such as low efficiency and high error rates. Therefore, improving the detection accuracy of CT images and reducing the misdiagnosis rate remain major challenges that need to be addressed urgently in the medical field. With the rapid development of deep learning technology, it provides new ideas for improving the detection accuracy and efficiency of CT images. This study proposes a Dual-Path Network (DPN) method based on deep learning, aiming to construct an efficient and accurate brain tumor CT image anomaly detection system. The core idea of the DPN model is to integrate the key structures of the ResNet model and the DenseNet model, including the bottleneck structure from ResNet and the grouped convolution strategy from DenseNet. By combining these structures, the advantages of both models are fully utilized, while effectively addressing the issue of high computational overhead that may arise from using a large number of parameters. This research successfully constructs a brain tumor anomaly detection system by leveraging the DPN model's characteristics of precise detection and reduced resource consumption. The system aims to accelerate the application of deep learning technology in the medical field, provide reliable decision support for clinicians and promote the further development of modern medicine.

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Structure-Aware and Consistency-Preserved Graph Contrastive Learning without Augmentation

Haoxuan Zhang, Yafang Li, Jianqiang Li
Beijing University of Technology, China

Graph contrastive learning (GCL) has made significant progress in unsupervised graph representation learning. However, most methods rely on manually designed augmentations, which introduce high computational overhead and risk semantic

inconsistency-especially when perturbations distort graph structure or corrupt key features. To overcome these issues, we propose SCOPE (Structure-Aware and Consistency-Preserved graph contrastive learning), an augmentation-free framework that exploits intrinsic graph information to define meaningful contrastive objectives.

Specifically, we propose a structure-aware positive sampling strategy using partial absorption scores to select topologically similar nodes as positives, ensuring semantic relevance without artificial noise. Meanwhile, a feature-driven KNN graph serves as an auxiliary view, and consistency between embeddings from the original and KNN graphs is enforced via a cross-view alignment loss. This dual approach removes the need for stochastic augmentations while preserving structural and attribute semantics. We evaluate SCOPE on six benchmark datasets, consistently achieving competitive or superior results compared to other contrastive learning methods. These results highlight the effectiveness of structure-aware sampling and consistency preservation in improving the stability and efficiency of contrastive learning on graphs.

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Bifurcation of Periodic Solutions for a Cross-scale Oscillator Chain Model

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Acoustic metamaterials achieve precise control of sound waves by artificially designing structures, providing new solutions for sound field stealth and acoustic super-resolution imaging. Since its energy transfer is closely related to the nonlinear equation of the oscillator chain, the study of the dynamics especially bifurcation of periodic solutions of the oscillator chain system is of great significance for the design of acoustic metamaterials and the non-reciprocity of energy transfer. In this paper, the bifurcation of periodic solutions of a cross-scale oscillator chain model with linear oscillator and cubic nonlinear oscillator is studied. By scale transformation, the oscillator chain model is transformed into a high-dimensional slow-fast non-autonomous system. Based on the Melnikov vector function, the existence conditions of periodic solutions of the system under periodic perturbation are investigated. Through numerical simulation, the number and geometric configuration of periodic solutions of the system under different parameter conditions are discussed. The research results have important theoretical significance for the parameter design of nonreciprocal metamaterials, and the theoretical foundation of vibration suppression and control is supported.

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Research on the Dynamics and Damping Performance of Parallel-Grounded Damping Nonlinear Energy Sink Cells

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Nonlinear energy sink (NES) has attracted considerable attention due to its advantages, including wide vibration reduction bandwidth and no need for additional energy consumption. However, for a primary system subjected to external excitation, the nonlinear mechanism of the NES requires the input energy to exceed a certain threshold to initiate energy transfer, and a single NES may be insufficient to meet the damping requirements of large-scale engineering structures. Therefore, this paper investigates the dynamics and vibration suppression performance of parallel-grounded damping NES cells under harmonic excitation. Taking connecting two cells as an example, the slow-flow and equilibrium point equations are derived by applying the complexification-averaging method. The boundary conditions for saddle-node bifurcations are obtained, and the stability of the steady-state solutions is analyzed. Moreover, the response characteristics within the resonance region are simulated using the fourth-order Runge-Kutta method. Finally, the damping performance of the proposed parallel configuration is compared with that of the series-configured NES cells under two different excitation conditions. Numerical results indicate that increasing the number of cells in the parallel configuration significantly enhances vibration attenuation efficiency, while for the series configuration, adding more cells can more effectively absorb the energy from the primary system, particularly when the initial displacement is large. The findings can offer theoretical guidance for the design of nonlinear vibration control devices in large-scale engineering structures.

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A Modified Model of the Transportation Task with Natural Disaster Risk Consideration

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Natural disasters such as floods, hurricanes, landslides, extreme rainfall, earthquakes have a serious impact on the sustainability of businesses related to transportation systems. There are interruptions or changes in transportation routes, delays in deliveries, increases in planned costs, disruptions in supply chains due to unforeseen changes in demand and supply, and other adverse events. The classical transportation task, which aims to minimize transportation costs under certain constant constraints (static costs and fixed capacities), does not take into account the uncertainty in the logistics system due to natural disasters. The purpose of this article is to propose a modified model of the classical transportation task, which includes an assessment of the risk of natural disasters as a key parameter in the optimization of logistics systems. The proposed modified model extends the classical transportation task by taking into account factors related to natural disasters: the level of potential risk along different routes, changes in supply chain capacity and demand levels.This modified model includes several additional parameters such as the risk level and probability of occurrence of a natural disaster, disaster intensity, infrastructure sensitivity, and route vulnerability. These additional parameters are used to assess the accessibility of the routes and dynamically recalculate the values in the model. The proposed extended model of the transport task allows to minimize the total transport costs, taking into account both standard logistical constraints and various risk parameters associated with potential natural disasters along the transport routes. Numerical results are presented, which show the behavior of the modified model of the transport task at different levels of risk of natural disasters.

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